Sequence of Grade 4 Modules Aligned with the Standards

Module 1: Place Value, Rounding, and Algorithms for Addition and Subtraction
Module 2: Unit Conversions and Problem Solving with Metric Measurement
Module 3: Multi-Digit Multiplication and Division
Module 4: Angle Measure and Plane Figures
Module 5: Fraction Equivalence, Ordering, and Operations
Module 6: Decimal Fractions
Module 7: Exploring Measurement with Multiplication

Summary of Year

Grade 4 mathematics is about (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Key Areas of Focus for 3–5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving

Required Fluency: 4.NBT.4 Add and subtract within 1,000,000.

Major Emphasis Clusters
Operations and Algebraic Thinking • Use the four operations with whole numbers to solve problems. Number and Operations in Base Ten • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. Number and Operations—Fractions • Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimal fractions.

Rationale for Module Sequence in Grade 4

In Grade 4, students extend their work with whole numbers. They begin with large numbers using familiar units (tens and hundreds) and develop their understanding of thousands by building knowledge of the pattern of *times ten* in the base-ten system on the place value chart (**4.NBT.1**). In

Grades 2 and 3, students focused on developing the concept of composing and decomposing place value units within the addition and subtraction algorithms. Now, in Grade 4, those (de)compositions are seen through the lens of multiplicative comparison (e.g., 1 thousand is 10 times as much as 1 hundred). They next apply their broadened understanding of patterns on the place value chart to compare, round, add, and subtract. The addition and subtraction algorithms are then efficient and useful applications of students' knowledge of and skill with composing and decomposing higher value units. The module culminates with solving multi-step word problems involving addition and subtraction modeled with tape diagrams that focus on numerical relationships.

The algorithms continue to play a part in Module 2 as students relate place value units to metric units. This module helps students draw similarities between:

1 ten	= 10 ones
1 hundred	= 10 tens
1 hundred	= 100 ones
1 meter	= 100 centimeters
1 thousand	= 1,000 ones
1 kilometer	= 1,000 meters
1 kilogram	= 1,000 grams
1 liter	= 1,000 milliliters

Students work with metric measurement in the context of the addition and subtraction algorithms, mental math, place value, and word problems. Customary units are used as a context for fractions in Modules 5 and 7.

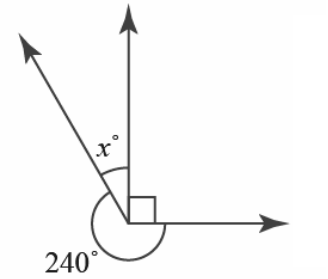
In Module 3, measurement of perimeter and area provide the concrete foundation behind the distributive property in the multiplication algorithm: 4 × (1 m 2 cm) can be modeled concretely using ribbon, since it is easy to see the 4 copies of 1 meter and the 4 copies of 2 centimeters. Likewise, 4 × (1 ten 2 ones) = 4 tens 8 ones. Students next use place value disks to develop efficient procedures and the algorithms for multiplying and dividing one-digit whole numbers. They understand and explain why the procedures work, and connections are made between the area model and work on the place value chart. Two-digit by two-digit multiplication is then modeled using the area model, extending students' earlier experiences with measurement and the distributive property. Students also solve word problems throughout the module where they select and accurately apply appropriate methods to estimate, mentally calculate, or use written strategies to compute products and quotients.

Module 4 focuses as much on solving unknown angle problems using letters and equations as it does on building, drawing, and analyzing two-dimensional shapes in geometry. Students have already used letters and equations to solve word problems in earlier grades. They continue to do so in Grade 4, and now they also learn to solve unknown angle problems: work that challenges students to build and solve equations to find unknown angle measures. First, students learn the definition of degree and learn how to measure angles in degrees using a circular protractor. From the definition of degree and the fact that angle measures are additive, the following rudimentary facts about angles naturally follow:

1. The sum of angle measurements around a point is 360 degrees.
2. The sum of angle measurements on a line is 180 degrees.

Hence, from 1 and 2, students see that vertical angles are equal. Armed only with these facts, students are able to generate and solve equations as in the following problem:

Find the unknown angle x.



$x + 240 + 90 = 360$

$x + 330 = 360$

$x = 30$

The unknown angle is 30 degrees.

Unknown angle problems help to unlock algebraic concepts for students because such problems are visual. The x clearly stands for a specific number. If a student wished, he could place a protractor down on that angle and measure it to find x. But doing so destroys the joy of deducing the answer and solving the puzzle on his own.

Module 5 centers on equivalent fractions and operations with fractions. We use fractions when there is a given unit, the *whole unit*, but we want to measure using a smaller unit, called the *fractional unit*. To prepare students to explore the relationship between a fractional unit and its whole unit, examples of such relationships in different contexts were already carefully established earlier in the year:

360 degrees in	1 complete turn
100 centimeters in	1 meter
1000 grams in	1 kilogram
1000 milliliters in	1 liter

The beauty of fractional units, once defined and understood, is that they behave just as all other units do:

- "3 fourths + 5 fourths = 8 fourths" just as "3 meters + 5 meters = 8 meters"
- "4 × 3 fourths = 12 fourths" just as "4 × 3 meters = 12 meters"

Students add and subtract fractions with like units using the area model and the number line. They multiply a fraction by a whole number where the interpretation is as repeated addition (e.g., 3 fourths + 3 fourths = 2 × 3 fourths). Through this introduction to fraction arithmetic they gradually come to understand fractions as units they can manipulate, just like whole numbers. Throughout the module, customary units of measurement provide a relevant context for the arithmetic.

Module 6, on decimal fractions, starts with the realization that decimal place value units are simply special fractional units: 1 tenth = 1/10, 1 hundredth = 1/100, etc. Fluency plays an important role in this topic as students learn to relate 3/10 = 0.3 = 3 tenths. They also recognize that 3 tenths is equal to 30 hundredths and subsequently have their first experience adding and subtracting fractions with unlike units (e.g., 3 tenths + 4 hundredths = 30 hundredths + 4 hundredths).

The year ends with a module focused on multiplication and measurement, as they solve multi-step word problems. Exploratory lessons support conceptual understanding of the relative sizes of measurement units. Students explore conversion in hands-on settings and subsequently apply those conversions to solve multi-step word problems involving all operations and multiplicative comparison.

Alignment Chart[65]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
Module 1: **Place Value, Rounding, and Algorithms for Addition and Subtraction** (25 days)	**Use the four operations with whole numbers to solve problems.**[66] **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

[65] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.
[66] The balance of this cluster is addressed in Modules 3 and 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Generalize place value understanding for multi-digit whole numbers. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.) **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place. **Use place value understanding and properties of operations to perform multi-digit arithmetic.**[67] **4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Module 2: **Unit Conversions and Problem Solving with Metric Measurement** (7 days)	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[68] **4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …* **4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

[67] From this point forward, fluency practice is part of students' on-going experience. The balance of this cluster is addressed in Module 3.
[68] The focus of this module is on the metric system to reinforce place value, mixed units, and word problems with unit conversions. Decimal and fraction word problems wait until Modules 6 and 7. 4.MD.3 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
Module 3: **Multi-Digit Multiplication and Division** (43 days)	**Use the four operations with whole numbers to solve problems.** **4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Standards Glossary, Table 2.) **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Gain familiarity with factors and multiplies.** **4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)**[69] **4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the

[69] 4.NBT.4 is addressed in Module 1 and is then reinforced throughout the year.

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	relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[70] **4.MD.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
Module 4: **Angle Measure and Plane Figures** (20 days)	**Geometric measurement: understand concepts of angle and measure angles.** **4.MD.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

[70] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.2 is addressed in Modules 2, 6, and 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Draw and identify lines and angles, and classify shapes by properties of their lines and angles. **4.G.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **4.G.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Module 5: **Fraction Equivalence, Ordering, and Operations**[71] (45 days)	**Generate and analyze patterns.** **4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **Extend understanding of fraction equivalence and ordering. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)** **4.NF.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole.

[71] Tenths and hundredths are important fractions in this module, represented in decimal form in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.** **4.NF.3** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

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	c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Represent and interpret data.** **4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Module 6: **Decimal Fractions** (20 days)	**Understand decimal notation for fractions, and compare decimal fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)**[72] **4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **4.NF.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

[72] In this module, we continue to work with fractions, now including decimal form.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[73] **4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Module 7: **Exploring Measurement with Multiplication** (20 days)	**Use the four operations with whole numbers to solve problems.** **4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Standards Glossary, Table 2.) **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[74] **4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a

[73] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.3 is addressed in Module 3.
[74] The focus now is on customary units in word problems for application of fraction concepts. 4.MD.3 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules	
		larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …*
	4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.